EFFECT OF NONUNIFORM DISCHARGE ON THE EFFICIENCY OF HEAT EXCHANGERS

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A criterion of nonuniformity is proposed for the analysis of heat exchangers in which one of the heat carriers is nonuniformly distributed. Generalized quantitative relations are derived which make it possible to estimate the decrease of the regeneration factor. The basic results are verified by an experimental study.

Heat exchangers are, as a rule, used under conditions where the heat carriers are nonuniformly distributed among the ducts in the block and this is in many cases related to a considerable lowering of their performance indices [1, 2, 3]. There are a great many different manners in which the nonuniformity can vary, which makes the analysis and the design of such apparatus difficult while the feasibility of generalizing the results remains thus considerably limited.

These difficulties can be largely overcome by replacing the actual velocity profile with a linear relation. Such a transformation is permissible only insofar as the discharge nonuniformity criterion, which appears to have a definite effect on the change in the heat-exchanger performance indices, is maintained constant.

We will present the results of a heat-exchanger efficiency analysis for the case where one of the heat carriers is nonuniformly distributed.

In choosing the nonuniformity criterion we consider first the possibility of linearizing the velocity profile $\overline{G}_g$ in heat exchangers of the counterflow type (Fig. 1). In such a heat exchanger the individual elementary sections dx operate as completely independent units. For this reason, we may alter their relative spatial position while maintaining a constant regeneration factor ($\eta_r^* = \text{const}$). At the same time, however, the relative spacing of sections in the $\overline{G}_g$ profile will also change.

All these considerations predetermine the premises under which a profile $\overline{G}_g$ will be transformed invariantly with respect to η_r^* (Fig. 1). By way of such a transformation one can reduce all the diverse $\overline{G}_g$ profiles encountered in practice (curve 1) to a class of monotonically varying functions (curve 2). The latter can then be more or less closely approximated by linear relations (curve 3).

Inasmuch as the effect of discharge nonuniformity on the heat-exchanger efficiency depends not only on the degree of nonuniformity but also on the length of the section it covers, a parameter combining both these factors would be very convenient to use in the linearization of profile $\overline{G}_g$. One possible choice for such a parameter could be the mean-integral value of nonuniformity:

$$s = 0.5 \int_0^1 |\overline{G}_g - 1|\, dx. \tag{1}$$

The deviation of the monotonically varying profile from the linear relation approximating it can be characterized by the parameter s_1, which is essentially analogous to the parameter s:

$$s_1 = 0.5 \int_0^1 \left|\overline{G}_g - [1 + a(2x - 1)]\right| dx$$

or in relative form

$$\delta s_1 = (s_1/s)\ \ 100\%. \tag{2}$$

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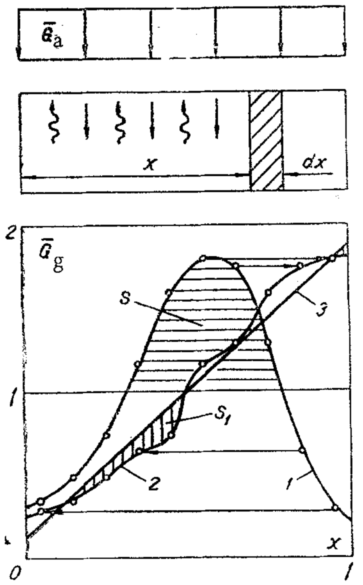

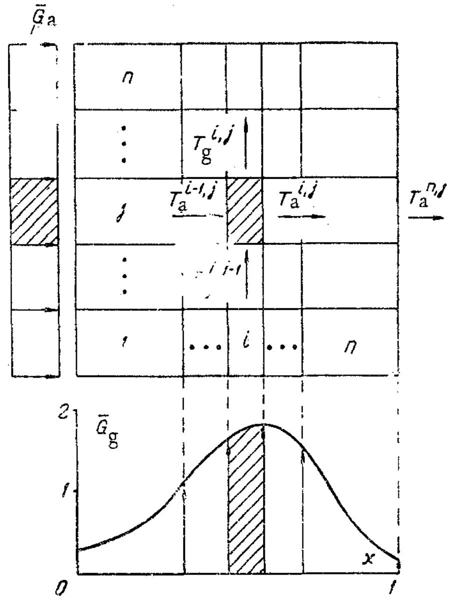

Fig. 1 Fig. 2

Fig. 1. Schematic diagram of a counterflow heat exchanger: 1) original profile $\bar{G}_g$; 2) monotonically varying profile $\bar{G}_g$; 3) linear profile $G_g = 1 + a(2x - 1)$.

Fig. 2. Schematic diagram of a crossflow heat exchanger.

If after linearization of any arbitrary $\bar{G}_g$ profile $\delta s_1 = 0$ as a result, then the said transformations have been entirely valid. If $\delta s_1 > 0$ as a result, however, then the linear $\bar{G}_g$ profile only approximately represents the effect of discharge nonuniformity on the thermal efficiency of the apparatus.

In order to estimate the level of δs_1 values permissible in any linearization process, the thermal efficiencies of a counterflow heat exchanger were calculated for a wide range of parameter variations ($\eta_r = 0.5$–0.9, $s = 0$–0.25) and assuming two different gas discharge laws: a) a stepwise (original) discharge; b) a linear discharge obtained by linearization of the original $\bar{G}_g$ profile.

As it turned out, the replacement of a stepped $\bar{G}_g$ profile by a linear relation (under the condition that s = const for both functions) yields a relatively small error in determining the magnitude of $\Delta\eta$. Thus, with $\delta s_1 = 25\%$ the maximum error in calculating $\Delta\eta$ for all the cases considered here was less than 17%, while with $\delta s_1 = 10\%$ it was less than 3%.

The results of these calculations show that the determining factor in lowering the thermal efficiency of a heat exchanger is the mean-integral value of the nonuniformity and, consequently, this parameter may be used as the criterion of nonuniformity.

The law according to which the nonuniformity varies across the front surface of a heat exchanger is of lesser significance here and, therefore, the replacement of any arbitrary $\bar{G}_g$ profile by a linear relation is in many cases permissible, provided that the nonuniformity criterion is maintained the same (s = const) for both functions. A similar criterion of nonuniformity can be used in the linearization of the velocity profile in heat exchangers of the crossflow type. The feasibility of transforming profile $\bar{G}_g$ invariantly with respect to η_r^* was in our case demonstrated as follows.

The heat exchanger (Fig. 2) was subdivided into n^2 elementary sections in such a way that the water equivalent ratio remained the same for the heat carrier in each section and for the total unit ($\omega_{i,j} = \omega$). For the $N_{i,j}$-th section we may then write:

$$T_a^{i,j} = T_a^{i-1,j} + \eta_i \left(T_g^{i,j-1} - T_a^{i-1,j}\right), \tag{3}$$

$$T_g^{i,j} = T_g^{i,j-1} - \frac{\eta_i}{\omega}\left(T_g^{i,j-1} - T_a^{i-1,j}\right). \tag{4}$$

On the basis of expressions (3) and (4), the method of mathematical induction was now applied to obtain expressions for the heat carrier temperatures at the exit of the $N_{i,j}$-th section:

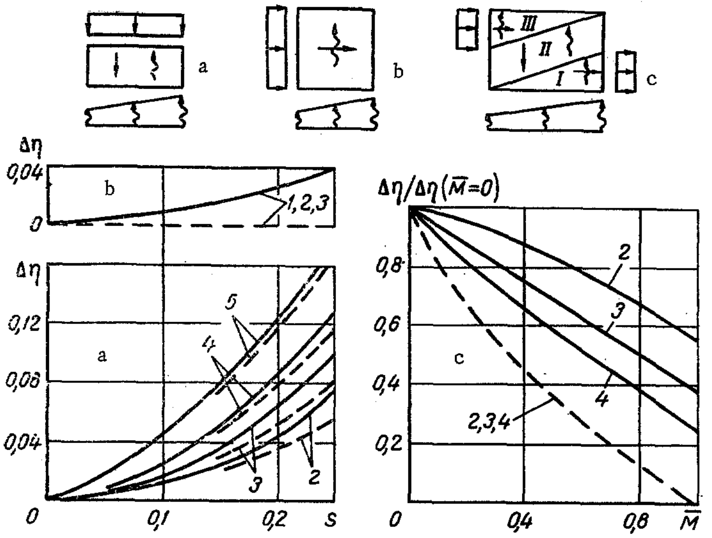

Fig. 3. Variation of the regeneration factor as a function of the mean-integral discharge nonuniformity (for $\omega = 1$): 1) $\eta_r = 0.5$; 2) $\eta_r = 0.6$; 3) $\eta_r = 0.7$; 4) $\eta_r = 0.8$; 5) $\eta_r = 0.9$; solid line) c = 0; dashed line) c = 1. a) Counterflow; b) crossflow; c) Z-flow.

$$T_g^{i,j} = \sum_{k=0}^{j-1}\left\{\frac{(j-1)!}{k!\,(j-k-1)!}\,\frac{1}{\omega^k}\sum_{l=1}^{i}\left[-(-1)^{l+k}\sum\left(C_i^l\sum\Gamma_l^k\right)\right]\right\}, \quad (5)$$

$$T_g^{i,j} = \sum_{k=0}^{j}\left\{\frac{j!}{k!\,(j-k)!}\,\frac{1}{\omega^k}\left\{\sum_{l=1}^{i-1}\left[-(-1)^{l+k}\sum\left(C_{i-1}^l\sum\Gamma_l^k\right)\right] + \sum_{l=1}^{i}\left[-(-1)^{l+k}\sum\left(C_{i-1}^{l-1}\sum\Gamma_l^k\right)\right]\right\}\right\}. \quad (6)$$

The symbols used in these formulas have the following meaning: $\sum C_i^l$ is the sum of all products combining l-factors out of i elements ($\eta_1, \eta_2, \eta_3, \ldots, \eta_i$) without repetitions, $C_i^l \sum \Gamma_l^k$ is the product of one combination (product) C_i^l by the sum of all products Γ_l^k combining l-factors (contained in the combination C_i^l) out of k elements with repetitions, $C_{i-1}^{l-1} \sum \Gamma_l^k$ is the product of one combination (product) C_{i-1}^{l-1} by the sum of all products Γ_l^k combining l-factors (of which element $l-1$ is contained in the combination C_{i-1}^{l-1}, and another element η_i) out of k elements with repetitions. It is also assumed that $\Gamma_l^0 = C_{i-1}^0 = 1$.

The meaning of these symbols can be illustrated by the following examples:

$$\sum C_i^l = \sum C_3^2 = \eta_1\eta_2 + \eta_1\eta_3 + \eta_2\eta_3,$$

$$\sum\left(C_i^l\sum\Gamma_l^k\right) = \sum\left(C_3^2\sum\Gamma_2^2\right) = \eta_1\eta_2(\eta_1^2 + \eta_2^2 + \eta_1\eta_2) + \eta_1\eta_3(\eta_1^2 + \eta_3^2 + \eta_1\eta_3) + \eta_2\eta_3(\eta_2^2 + \eta_3^2 + \eta_2\eta_3),$$

$$\sum C_{i-1}^{l-1} = \sum C_2^1 = \eta_1 + \eta_2,$$

$$\sum\left(C_{i-1}^{l-1}\sum\Gamma_l^k\right) = \sum\left(C_2^1\sum\Gamma_2^2\right) = \eta_1(\eta_1^2 + \eta_i^2 + \eta_1\eta_i) + \eta_2(\eta_2^2 + \eta_i^2 + \eta_2\eta_i).$$

If the values i = n are inserted into Eq. (5), a relation will be obtained for calculating the air temperatures $T_a^{n,j}$ at the exit from the heat exchanger. An analysis of this relation has led to the following conclusions:

1. The η_i values for the individual elementary sections enter into the expression for $T_a^{n,j}$ as the sum of products (combinations) C_n^l.

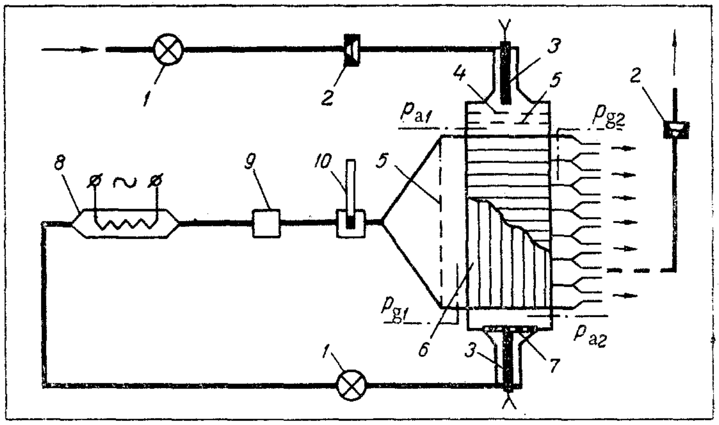

Fig. 4. Schematic diagram of the experimental setup: 1) valve; 2) measuring diaphragm; 3) resistance thermometer; 4) paranite packing seal with holes for equalizing the velocity field; 5) grid; 6) heat exchanger; 7) copper plate with holes for equalizing the temperature field; 8) electric heater; 9) mixer; 10) mercury thermometer.

2. All the η_i values are used in forming these products (combinations).

In this way, the expressions for $T_a^{n,j}$ and, consequently, also for the regeneration factor η_r^* are symmetric with respect to the value of $\eta_i = f(\bar{G}_{gi})$ as well as independent of the spatial distribution of sections. This then establishes the feasibility of transforming a $\bar{G}_g$ profile in crossflow heat exchangers invariantly with respect to η_r^*. Furthermore, such a transformation, when applied to the case of a heat carrrier uniformly distributed among the ducts in the block, leaves the temperature profile at the heat-exchanger exit also invariant.

A numerical analysis was performed with the following assumptions.

The discharge nonuniformity was given by the linear relation $\bar{G}_g = 1 + a(2x - 1)$ and the concurrent variation of the local heat-transfer coefficient by the relation $k_x = k\bar{G}_g^c$.

By substituting $k_x = f(\alpha_{gx}, \alpha_a)$ and $k = f(\alpha_g, \alpha_a)$ we obtained an expression for $c = f(\bar{G}_g, \alpha_g, \alpha_a)$. In order to simplify the calculation, the slight dependence of c across the block face on $\bar{G}_g$ was disregarded and the value of c was taken as equal throughout to that for $\bar{G}_g = 1$:

$$c = \frac{n_r}{1 + \alpha_g F_g / \alpha_a F_a},$$

where n_g is the exponent of Re_g in the relation $Nu_g = A_g Re_g^{n_g}$.

The relative change in the gas temperature $\eta_{gi} = (t_{g1} - t_{g2}^i)/(t_{g1} - t_{a1})$ at the N_i-th elementary section of a counterflow heat exchanger was determined by the equation [4]:

$$\eta_{gi} = \begin{cases} M_{gi}/(1 + M_{gi}) & \text{for } \omega_i = 1, \\ \dfrac{1 - \exp[(\omega_i - 1) M_{gi}]}{1 - \omega_i \exp[(\omega_i - 1) M_{gi}]} & \text{for } \omega_i \neq 1, \end{cases}$$

with $\omega_i = \bar{G}_{gi}\omega$ and $M_{gi} = kF/c_{rg}G_g\bar{G}_{gi}^{1-c}$.

The analogous relation $\eta_{gi} = (t_g^{i,j-1} - t_g^{i,j})/(t_g^{i,j-1} - t_a^{i-1,j})$ for the $N_{i,j}$-th elementary section of a crossflow heat exchanger (Fig. 2) was taken from [5]: $\eta_{gi} = M_{gi}/[1 + 0.5(1 + \omega_i)M_{gi}]$ with $\omega_i = G_{gi}\omega$ and $M_{gi} = kF/c_{rg}G_g n\bar{G}_{gi}^{1-c}$. The error in calculating η_{gi} is less than 1%, according to the estimate made by the author of [5], if M_{gi} and $\omega_i M_{gi}$ are both less than 0.4.

The mean-mass temperature of the gas at the exit was determined from $t_{g2} = \sum_{i=1}^{n} \bar{G}_{gi} t_{g2}^i$ for a counterflow heat exchanger and from $t_{g2} = \sum_{i=1}^{n} \bar{G}_{gi} t_g^{i,n}$ for a crossflow heat exchanger, the regeneration factor was calculated from the formula

$$\eta_r^* = \begin{cases} (t_{g1} - t_{g2})/(t_{g1} - t_{a1}) & \text{for} \quad \omega \leqslant 1, \\ \omega (t_{g1} - t_{g2})/(t_{g1} - t_{a1}) & \text{for} \quad \omega \geqslant 1. \end{cases}$$

The results of these computer-aided calculations are shown in Fig. 3.

The degree to which the discharge nonuniformity influences the heat-exchanger efficiency depends on the pattern of heat carriers movement, on the rated value of the regeneration factor, and on the heat-transfer surface characteristics.

The effect of nonuniformity is most pronounced in counterflow heat exchangers (Fig. 3a). In units of this type the heat transfer between individual gas streamers and the air over the entire extent of the ducts occurs with the water equivalents ratios for the heat carriers remaining constant. When the discharge is nonuniform, this causes the medium with a lower water equivalent to heat (cool) fast already in the section near the entrance. As a result, the temperature drop in the remaining part of the ducts will be considerably reduced and a large portion of the surface becomes not fully effective.

In crossflow heat exchangers the air streamers (heat carriers uniformly distributed among the ducts in the block) move with a continually changing water equivalents ratio ω_i, but the mean value of ω for the entire path can be calculated. Under such conditions, the slower heating of air at some surface segments (where $\omega_i < \omega$) is partially compensated by a faster heating at other surface segments (where $\omega_i > \omega$) and this leads to a much less reduced efficiency than in the case of counterflow heat exchangers.

The effect of the rated η_r and c values is to be explained as due to similar factors.

In heat exchangers designed with complex patterns of heat carrier flow (multipath-crossflow, Z-flow) it is also of secondary significance how the discharge nonuniformity varies across the block face. This was confirmed by efficiency calculations for such heat exchangers with the heat carriers distributed according to either $\overline{G}_g = 1 + a(2x - 1)$ or $\overline{G}_g = 1 - a(2x - 1)$: the values of $\Delta\eta$ for both cases did not differ by more than 0.003-0.004.

In multipath-crossflow heat exchangers the effect of the discharge nonuniformity on the regeneration factor appears about equally independent of the number of sections and, therefore, the magnitudes of $\Delta\eta$ here may be estimated from the data of Fig. 3b for a single-path crossflow.

In Z-flow heat exchangers the change of the regeneration factor may be estimated from the data of Fig. 3c. Here $\overline{M}$ is a parameter which accounts for the effect of sections with the working media in a crossflow on the value of η_r:

$$\overline{M} = \frac{2M_g'}{2M_g' + M_g''}, \text{ where } M_g' = \frac{k'F'}{c_{rg}G_g} \text{ and } M_g'' = \frac{k''F''}{c_{rg}G_g}.$$

Here symbols with a prime refer to sections I and III of the heat exchanger (Fig. 3c) with a crossflow of the working media, while symbols with a double prime refer to section II with a counterflow.

The Z-flow heat exchanger becomes a simple counterflow unit when $\overline{M} = 0$ and a single-path crossflow unit with $\overline{M} = 1$.

Experiments were performed with a setup shown schematically in Fig. 4.

The heat exchangers Nos. 1 and 2 used in testing were of a plate-and-rib construction with approximately triangular cross-section ducts and smooth walls. Unit No. 1 was arranged to produce a crossflow pattern, unit No. 2 differed from it only in the mode of the heat carrier movement: the ducts in the air compartment were arranged into a Z-pattern with the flow deflected twice by 90° (Fig. 3c). Provisions were also made for rearranging unit No. 1 to produce a double-path flow (unit No. 1a).

The thermal characteristics of the useful heating surfaces were determined on a special test stand for the range of Reynolds numbers from 200 to 1000 approximated by the relation $Nu = 566\ Re^{0.29}$.

The discharge nonuniformity was effected by sectionalizing the "gas" stream at the exit from the heat-transfer block and by subsequently throttling the "gas" flow unequally with a special device (Fig. 4). The "gas" discharge rates G_{gi} were measured with a calibrated diaphragm successsively inserted into each of the seven sections in the gas compartment.

TABLE 1. Results of Heat-Exchanger Tests with a Nonuniform Distribution of the "Gas"

Heat exchanger No.	$\overline{M}$	G, kg/h	η_r	$\overline{G}_{g1}$ ($\Delta x_i = 1/7$) 1	2	3	4	5	6	7	s	δs_i, %	c	$\Delta\eta_{exp.}$	$\Delta\eta_{calc.}$
1			0,682	1,01	1,01	1,01	0,99	0,985	0,99	1,005	—	—	—	—	—
	1	15,1	—	0,305	0,542	0,749	0,984	1,219	1,144	1,76	0,203	10	0,15	0,018	0,021
			—	1,805	1,43	1,195	0,978	0,752	0,538	0,303	0,204	10	0,15	0,020	0,021
			0,628	1,005	1,028	0,988	1,0	0,984	0,997	0,997	—	—	—	—	—
	1	25,2	—	0,296	0,585	0,778	0,985	1,21	1,55	1,60	0,194	10	0,15	0,021	0,020
			—	1,638	1,575	1,21	0,940	0,771	0,567	0,298	0,204	10	0,15	0,019	0,022
			0,740	1,006	0,988	0,988	1,0	1,006	1,006	1,006	—	—	—	—	—
1a	—	14,9	—	0,210	0,446	0,802	1,01	1,295	1,49	1,74	0,220	10	0,17	0,019	0,024
			—	1,63	1,50	1,27	1,035	0,793	0,552	0,22	0,205	10	0,17	0,021	0,021
2			0,742	1,035	0,995	1,0	1,0	1,0	0,965	1,005	—	—	—	—	—
	0,36	14,8	—	0,308	0,558	0,784	1,0	1,245	1,483	1,653	0,192	10	0,16	0,035	0,043
			—	1,644	1,495	1,197	1,003	0,784	0,558	0,309	0,192	10	0,16	0,032	0,043
	0,39	25,4	0,687	0,992	1,0	1,022	0,992	1,022	1,0	0,972	—	—	—	—	—
			—	0,304	0,565	0,795	1,02	1,21	1,49	1,62	0,191	10	0,17	0,030	0,0365
			—	1,60	1,50	1,22	1,005	0,81	0,575	0,291	0,188	10	0,17	0,027	0,0355

The air temperatures t_{a1} and t_{a2} were measured with TUÉ-40 nickel resistance thermometers and a UMV bridge giving indications with a 0.2° precision; the "gas" temperature was measured with a mercury thermometer having 0.1° divisions.

The experimental data were then processed according to the formulas:

$$\overline{G}_{gi} = G_{gi} \Big/ \sum_{i=1}^{7} G_{gi}$$

and

$$\Delta\eta = \left.\frac{t_{a2} - t_{a1}}{t_{g1} - t_{a1}}\right|_{\overline{G}_{gi}=1} - \left.\frac{t_{a2} - t_{a1}}{t_{g1} - t_{a1}}\right|_{\overline{G}_{gi}\neq 1}.$$

Tests were performed under the following conditions: $G \approx 15$ kg/h, $t_{g1} \approx 410$°K, $t_{a1} \approx 293$°K, $p_{a1} \approx 15 \cdot 10^4$ N/m², $p_{g1} \approx 11 \cdot 10^4$ N/m², and for heat exchangers Nos. 1 and 2 at $G \approx 25$ kg/h and $p_{g1} \approx 12 \cdot 10^4$ N/m².

For the crossflow heat exchangers Nos. 1 and 1a the agreement between experimental and calculated values of $\Delta\eta$ (see Table 1) was found satisfactory. Also the fact that the values of $\Delta\eta$ in such heat exchangers do not depend much on the rated regeneration factor and on the number of sections was confirmed by this experiment.

Experimental values of $\Delta\eta$ fall somewhat below calculated ones for the Z-flow heat exchanger No. 2 and the explanation for this is, evidently, that the effect of an error in determining the various parameters (s, δs_1, c, etc.) is greater in the counterflow situation.

On the whole, the performed tests confirm the conclusions arrived at by calculation and theoretical analysis, namely:

The mean-integral nonuniformity s has the predominant effect on any change in the regeneration factor and, therefore, it may be used as the criterion of nonuniformity. The manner in which the nonuniformity varies is much less significant.

The effect of the discharge nonuniformity on the regeneration factor of a heat exchanger is most pronounced when the heat carriers move in a counterflow, especially at high values of η_r.

In crossflow heat exchangers (regardless of the number of sections) the nonuniformity of discharge results in a much less reduced thermal efficiency and, moreover, the effect of the rated η_r value is insignificant here.

In all cases, heat-transfer elements operating with a turbulent flow of the working medium will improve the performance indices whenever the heat carriers are nonuniformly distributed.

NOTATION

$\overline{G}_g = G_{gx}/G_{g,mean}$	is the discharge nonuniformity;
G_{gx}, $G_{g,mean}$	are the local and mean discharge rates per unit of heat-exchanger width;
G_g, G_a	are the total discharge rates of heat carriers (gas and air);
a	is the maximum value of linear nonuniformity $\overline{G}_g = 1 + a(2x - 1)$;
s	is the mean-integral nonuniformity;
s_1, $\delta s_1 = (s_1/s) \cdot 100\%$	are the mean-integral and relative deviations of a profile from the linear relation approximating it;
η_r, η_r^*	are the regeneration factor at uniform and at nonuniform distribution of heat carriers;
$\Delta\eta = \eta_r - \eta_r^*$	is the change in the regeneration factor;
η_{gi}	is the parameter characterizing the thermal efficiency of an elementary section;
$\omega = c_{rg}G_g/c_{ra}G_a$	is the ratio between the water equivalents of the heat carriers;
c_{rg}, c_{ra}	are the mean specific heat of the gas and of air;
c	is the parameter characterizing the variation of local heat-transfer coefficients: $k_x = kG_g^c$;
k, α_g, α_a	are the heat transmission and heat-transfer coefficients at uniform distribution of heat carriers;

t_{a1}, t_{g1}, t_{a2}, t_{g2}	are the mean-mass temperatures of heat carriers (air and gas) at the entrance to and at the exit from a heat exchanger;
$T = (t - t_{a1})/(t_{g1} - t_{a1})$	is the temperature of a heat carrier expressed as a dimensionless quantity;
n	is the number of elementary sections;
n_g	is the exponent in the expression $Nu_g = A_g Re^{n_g}$;
F	is the area of the heat-exchanger surface;
C_i^l	is the combinations (products) of l-factors out of i elements;
Γ_l^k	is the combinations (products) of k factors out of l-elements with repetitions.

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